Selecting the SureStep™ Stepping System

Appendix C

In This Appendix...

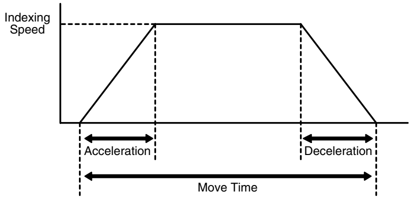
Selecting the SureStep™ Stepping System

The selection of your SureStep™ stepping system follows a defined process. Let's go through the process and define some useful relationships and equations. We will use this information to work some typical examples along the way.

The Selection Procedure

The motor provides for the required motion of the load through the actuator (mechanics that are between the motor shaft and the load or workpiece). Key information to accomplish the required motion is:

- total number of pulses from the PLC
- positioning resolution of the load
- indexing speed (or PLC pulse frequency) to achieve the move time
- required motor torque (including the 100% safety factor)
- load to motor inertia ratio



In the final analysis, we need to achieve the required motion with acceptable positioning accuracy.

How many pulses from the PLC to make the move?

The total number of pulses to make the entire move is expressed with the equation:

Equation ①: P_{total} = total pulses = (D_{total} ÷ (d_{load} ÷ i)) x θ_{step}

D_{total} = total move distance

d_{load} = lead or distance the load moves per revolution of the actuator's drive shaft
(**P** = pitch = 1/**d_{load}**)

θ_{step} = driver step resolution (steps/rev_{motor})

i = gear reduction ratio (rev_{motor}/$rev_{gearshaft}$)

Example 1: The motor is directly attached to a disk, the stepping driver is set at 400 steps per revolution and we need to move the disk 5.5 revolutions. How many pulses does the PLC need to send the driver?

P_{total} = (5.5 rev_{disk} ÷ (1 rev_{disk}/$rev_{driveshaft}$ ÷ 1 rev_{motor}/$rev_{driveshaft}$))
x 400 steps/rev_{motor}
= 2200 pulses

Example 2: The motor is directly attached to a ballscrew where one turn of the ballscrew results in 10 mm of linear motion, the stepping driver is set for 1000 steps per revolution, and we need to move 45 mm. How many pulses do we need to send the driver?

$\mathbf{P_{total}}$ = (45 mm ÷ (10 mm/rev$_{screw}$ ÷ 1 rev$_{motor}$/rev$_{screw}$)) x 1000 steps/rev$_{motor}$
= 4500 pulses

Example 3: Let's add a 2:1 belt reduction between the motor and ballscrew in example 2. Now how many pulses do we need to make the 45 mm move?

$\mathbf{P_{total}}$ = (45 mm ÷ (10mm/rev$_{screw}$ ÷ 2 rev$_{motor}$/rev$_{screw}$)) x 1000 steps/rev$_{motor}$
= 9000 pulses

What is the positioning resolution of the load?

We want to know how far the load will move for one pulse or step of the motor shaft. The equation to determine the positioning resolution is:

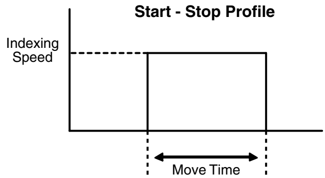
Equation ②: L_θ = load positioning resolution = (d_{load} ÷ i) ÷ θ_{step}

Example 4: What is the positioning resolution for the system in example 3?

$\mathbf{L_\theta}$ = ($\mathbf{d_{load}}$ ÷ **i**) ÷ θ_{step}
= (10 mm/rev$_{screw}$ ÷ 2 rev$_{motor}$/rev$_{screw}$) ÷ 1000 steps/rev$_{motor}$
= 0.005mm/step
≈ 0.0002in/step

What is the indexing speed to accomplish the move time?

The most basic type of motion profile is a "start-stop" profile where there is no acceleration or deceleration period. This type of motion profile is only used for low speed applications because the load is "jerked" from one speed to another and the stepping motor will stall or drop pulses if excessive speed changes are attempted. The equation to find indexing speed for "start-stop" motion is:



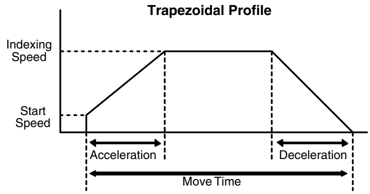
Equation ③: f_{SS} = indexing speed for start-stop profiles = P_{total} ÷ t_{total}

$\mathbf{t_{total}}$ = move time

Example 5: What is the indexing speed to make a "start-stop" move with 10,000 pulses in 800 ms?

f_{SS} = indexing speed = $P_{total} \div t_{total}$ = 10,000 pulses ÷ 0.8 seconds
= 12,500 Hz

For higher speed operation, the "trapezoidal" motion profile includes controlled acceleration & deceleration and an initial non-zero starting speed. With the acceleration and deceleration periods equally set, the indexing speed can be found using the equation:



Equation ④: $f_{TRAP} = (P_{total} - (f_{start} \times t_{ramp})) \div (t_{total} - t_{ramp})$
for trapezoidal motion profiles

f_{start} = starting speed
t_{ramp} = acceleration or deceleration time

Example 6: What is the required indexing speed to make a "trapezoidal" move in 800ms, accel/decel time of 200 ms each, 10,000 total pulses, and a starting speed of 40 Hz?

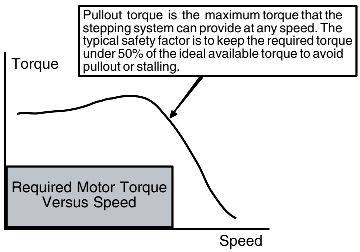
f_{TRAP} = (10,000 pulses - (40 pulses/sec x 0.2 sec)) ÷ (0.8 sec - 0.2 sec)
≈ 16,653 Hz

Calculating the Required Torque

The required torque from the stepping system is the sum of acceleration torque and the running torque. The equation for required motor torque is:

Equation ⑤: $T_{motor} = T_{accel} + T_{run}$

T_{accel} = motor torque required to accelerate and decelerate the total system inertia (including motor inertia)



T_{run} = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.

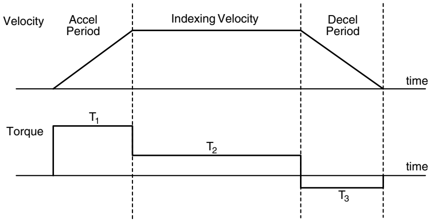
In **Table 1** we show how to calculate torque required to accelerate or decelerate an inertia from one speed to another and the calculation of running torque for common mechanical actuators.

Table 1 – Calculate the Torque for "Acceleration" and "Running"

The torque required to accelerate or decelerate an inertia with a linear change in velocity is:

Equation ⑥: $T_{accel} = J_{total}$ x (Δspeed ÷ Δtime) x (2π ÷ 60)

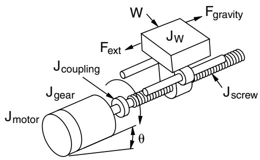
J_{total} is the motor inertia, plus load inertia ("reflected" to the motor shaft). The (2π ÷ 60) is a factor used to convert "change in speed" expressed in RPM into angular speed (radians/second). Refer to information in this table to calculate "reflected" load inertia for several common shapes and mechanical mechanisms.



Example 7: What is the required torque to accelerate an inertia of 0.002 lb·in·sec^2 (motor inertia is 0.0004 lb·in·sec^2 and "reflected" load inertia is 0.0016 lb·in·sec^2) from zero to 600 RPM in 50 ms?

T_{accel} = 0.002 lb·in·sec^2 x (600 RPM ÷ 0.05 seconds) x (2π ÷ 60)
≈ 2.5 lb·in

Leadscrew Equations



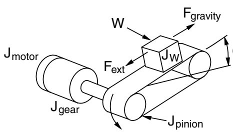
Description:	Equations:
Motor RPM	$n_{motor} = (v_{load} \times P) \times i$, n_{motor} (RPM), v_{load} (in/min)
Torque required to accelerate and decelerate the load	$T_{accel} \approx J_{total} \times (\Delta speed \div \Delta time) \times 0.1$
Motor total inertia	$J_{total} = J_{motor} + J_{gear} + ((J_{coupling} + J_{screw} + J_W) \div i^2)$
Inertia of the load	$J_W = (W \div (g \times e)) \times (1 \div 2 \pi P)^2$
Pitch and Efficiency	P = pitch = revs/inch of travel, e = efficiency
Running torque	$T_{run} = ((F_{total} \div (2 \pi P)) + T_{preload}) \div i$
Torque due to preload on the ballscrew	$T_{preload}$ = ballscrew nut preload to minimize backlash
Force total	$F_{total} = F_{ext} + F_{friction} + F_{gravity}$
Force of gravity and Force of friction	$F_{gravity} = W\sin\theta$, $F_{friction} = \mu W\cos\theta$
Incline angle and Coefficient of friction	θ = incline angle, μ = coefficient of friction

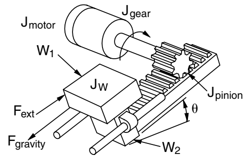
Table 1 (cont'd)

Typical Leadscrew Data

Material:	e = efficiency	Material:	μ = coef. of friction
ball nut	0.90	steel on steel	0.580
acme with plastic nut	0.65	steel on steel (lubricated)	0.150
acme with metal nut	0.40	teflon on steel	0.040
		ball bushing	0.003

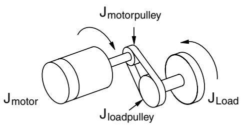
Belt Drive (or Rack & Pinion) Equations

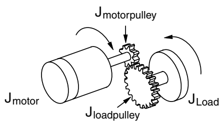




Description:	Equations:
Motor RPM	$n_{motor} = (v_{load} \times 2\pi r) \times i$
Torque required to accelerate and decelerate the load	$T_{accel} \approx J_{total} \times (\Delta speed \div \Delta time) \times 0.1$
Inertia of the load	$J_{total} = J_{motor} + J_{gear} + ((J_{pinion} + J_W) \div i^2)$
Inertia of the load	$J_W = (W \div (g \times e)) \times r^2$; $J_W = ((W_1 + W_2) \div (g \times e)) \times r^2$
Radius of pulleys	r = radius of pinion or pulleys (inch)
Running torque	$T_{run} = (F_{total} \times r) \div i$
Force total	$F_{total} = F_{ext} + F_{friction} + F_{gravity}$
Force of gravity and Force of friction	$F_{gravity} = W\sin\theta$; $F_{friction} = \mu W\cos\theta$

Belt (or Gear) Reducer Equations

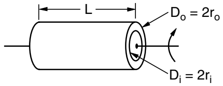




Description:	Equations:
Motor RPM	$n_{motor} = n_{load} \times i$
Torque required to accelerate and decelerate the load	$T_{accel} \approx J_{total} \times (\Delta speed \div \Delta time) \times 0.1$
Inertia of the load	$J_{total} = J_{motor} + J_{motorpulley} + ((J_{loadpulley} + J_{Load}) \div i^2)$
Motor torque	$T_{motor} \times i = T_{Load}$

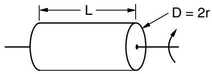
Table 1 (cont'd)

Inertia of Hollow Cylinder Equations



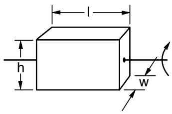
Description:	Equations:
Inertia	$J = (W \times (r_o^2 + r_i^2)) \div (2g)$
Inertia	$J = (\pi \times L \times \rho \times (r_o^4 - r_i^4)) \div (2g)$
Volume	$\text{volume} = \pi/4 \times (D_o^2 - D_i^2) \times L$

Inertia of Solid Cylinder Equations



Description:	Equations:
Inertia	$J = (W \times r^2) \div (2g)$
Inertia	$J = (\pi \times L \times \rho \times r^4) \div (2g)$
Volume	$\text{volume} = \pi \times r^2 \times L$

Inertia of Rectangular Block Equations



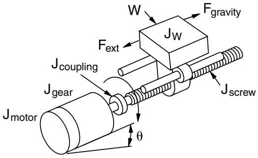
Description:	Equations:
Inertia	$J = (W \div 12g) \times (h^2 + w^2)$
Volume	$\text{volume} = l \times h \times w$

Symbol Definitions

J = inertia	ρ = density
L = Length	$\rho = 0.098\ lb/in^3$ (aluminum)
h = height	$\rho = 0.28\ lb/in^3$ (steel)
w = width	$\rho = 0.04\ lb/in^3$ (plastic)
W = weight	$\rho = 0.31\ lb/in^3$ (brass)
D = diameter	$\rho = 0.322\ lb/in^3$ (copper)
r = radius	
g = gravity = $386\ in/sec^2$	$\pi \approx 3.14$

Leadscrew – Example Calculations

Step 1 – Define the Actuator and Motion Requirements



Weight of table and workpiece = 200 lb, where W = 200 lb
Angle of inclination = 0°
Friction coefficient of sliding surfaces = 0.05, where μ = 0.05
External load force = 0
Ball screw shaft diameter = 0.6 inch
Ball screw length = 23.6 inch
Ball screw material = steel
Ball screw lead = 0.6 inch/rev, where P = 1/0.6 = 1.67 rev/in
Desired Resolution = 0.001 inch/step
Gear reducer = 2:1, where i = 2, preliminary (for the 3:1 example, i = 3)
Stroke = 4.5 inch
Move time = 1.7 seconds
Acceleration time = Deceleration time = 0.425 sec / 2 = 212.5 ms
Lead screw efficiency = 0.9, where e = 0.9
Coupling and gear reducer inertias are negligible, which are considered to be 0

Definitions
d_{load} = lead or distance the load moves per revolution of the actuator's drive shaft (P = pitch = 1/d_{load})
D_{total} = total move distance
θ_{step} = driver step resolution (steps/rev_{motor})
i = gear reduction ratio (rev_{motor}/$rev_{gearshaft}$)
T_{accel} = motor torque required to accelerate and decelerate the total system inertia (including motor inertia)
T_{run} = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.
t_{total} = move time
Start frequency = 20 Hz (as defined with a Module H0-CTRIO

Step 2 – Determine the Positioning Resolution of the Load

One revolution of the lead screw shaft advances 0.6 inches. We are looking for a 0.001 inch per step precision.

Check to see if 400 pulses/rev will achieve the desired precision:

With a reduction of 2:1, there will be two motor shaft revolutions to get a displacement of 0.6 inches.

(2 rev) x (400 pulses/rev) = 800 pulses for every 0.6 inches of displacement.

Therefore, (0.6 in) / (800 pulses) = 0.00075 in/pulse.

This is within the desired 0.001 in/step.

How many pulses are needed for the displacement?

With the 2:1 gear reduction, the stepping system can be set at 400 steps/rev to exceed the required load positioning resolution.

Since the lead screw advances 0.6 inches / rev, in the required stroke of 4.5 inches we will need:
(4.5 in) / (0.6 rev/in) = 7.5 revolutions on the lead screw

Since we have a reduction of 2:1, the motor shaft shall rotate 15 revolutions.

Step 3 – Determine the Motion Profile

Since we know that 400 pulses gets one revolution on the motor and we need 15 revolutions, then 400 x 15 = 6,000 pulses to move 4.5 inches.

From **Equation** ④, the indexing frequency for a trapezoidal move is:

$$\mathbf{f_{TRAP} = (P_{total} - (f_{start} \times t_{ramp})) \div (t_{total} - t_{ramp})}$$
$$= (6{,}000 - (20 \times 0.425s)) \div (1.7 - 0.425) = 4{,}699 \text{ Hz},$$
where the starting speed is 20 Hz

(4,699 Hz) / (400 steps/rev) = 11.7475 rev/s

To get it in rpm, (11.7475 rev/s) x (60 s/min) = 704.85 rpm.

Step 4 – Determine the Required Motor Torque

Using the equations in **Table 1**:
(Total inertia seen by the motor is the sum of all inertias)

$$J_{total} = J_{motor} + J_{gear} + ((J_{coupling} + J_{screw} + J_W) \div i^2)$$

For this example, we assume the gearbox and coupling inertias are zero.

Load inertia:

$$\mathbf{J_W = (W \div (g \times e)) \times (1 \div 2\pi P)^2}$$
$$= (200 \div (386 \times 0.9)) \times (1 \div 2 \times 3.1416 \times 1.67)^2$$
$$= 0.00523 \text{ lb·in·sec}^2$$

Lead screw inertia:

$$\mathbf{J_{screw} = (\pi \times L \times \rho \times r^4) \div (2g)}$$
$$= (3.1416 \times 23.6 \times 0.28 \times 0.3^4) \div (2 \times 386)$$
$$= 0.0002178 \text{ lb·in·sec}^2$$

The inertia of the load and screw reflected to the motor is the sum of both values divided by the square of the reduction ratio

$$\mathbf{J_{(screw + load)\ referred\ to\ motor} = ((J_{screw} + J_W) \div i^2)}$$

$$J_{\text{total less the motor inertia}} = ((0.0002178 + 0.00523) \div 2^2) = 0.001362 \text{ lb·in·sec}^2$$

The dynamic torque required to accelerate the inertia (without the motor rotor inertia) is:

$$\mathbf{T_{accel} = J_{total} \times ((\Delta_{rpm} \div \Delta_{time}) \times (2\pi \div 60))}$$
$$= 0.001362 \times ((704.85 \div 0.2125) \times (2 \times 3.1416 \div 60))$$
$$= 0.474309 \text{ lb·in}$$

Determine the running torque, or in this case the friction torque:

The forces are:

$$\mathbf{F_{total} = F_{ext} + F_{friction} + F_{gravity}}$$ [External forces and gravity are zero in this case.]
$$= 0 + (\mu \times W \times \cos\theta) + 0$$
$$= 0.05 \times 200 = 10 \text{ lb}$$

And the formula to be used is:

$$\mathbf{T_{run} = ((F_{total} \div (2\pi P)) + T_{preload}) \div i}$$
$$= (10 \div (2 \times 3.1416 \times 1.67)) \div 2$$ [related to the motor side]
$$= 0.4765 \text{ lb·in}$$ [where, we have assumed the preload torque to be zero]

From **Equation** ⑤, the minimum required motor torque @ 704.85 rpm is:

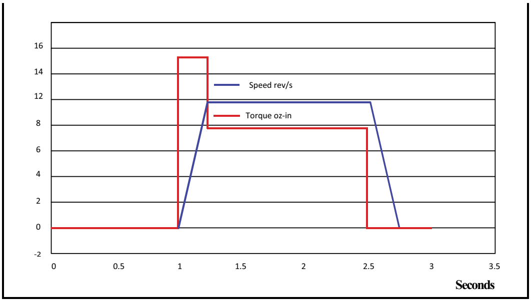
$$\mathbf{T_{motor} = T_{accel} + T_{run}}$$
$$= (0.474309 \text{ lb·in}) + (0.4765 \text{ lb·in})$$
$$= 0.9508 \text{ lb·in, or } 15.21 \text{ oz·in}$$

However, this is the required motor torque before we have picked a motor and included the motor inertia.

Step 5 – Select & Confirm the Stepping Motor & Driver System

There are two commonly used criteria to select a motor and drive:

- Take into account the calcluated torque. From step 4, we find we need 15.21 oz·in.
- Per a rule of thumb, the load to motor inertia ratio should be kept below 10.
- In step 4 we calculated that the $\mathbf{J_{(screw + load)}}$ = 0.001362 lb·in·sec^2. To find the ratio, we use the formula: $\mathbf{J_{(screw + load)} \div J_{motor}}$ The inertia of the motor is found in the motor specifications sheet.



We will check the criteria with 2 motors:

Figure 2 shows the Torque vs. Speed curves for STP-MTR-17040 and figure 3 shows STP-MTR-17048. We use this as an example to observe how different power supply voltages affect the torque output of a motor.

Consider STP-MTR-17040:

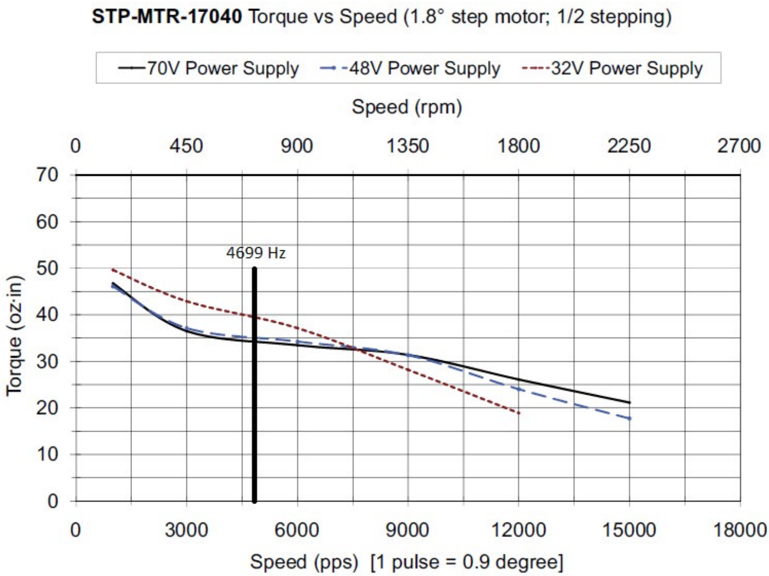


Figure 2: Torque for STP-MTR-17040 at 4.7 kHz

According to the torque/speed curves for this motor, the torque at 4.7 kHz is approximately 39 oz·in at 48VDC and 34 oz·in at 70VDC. Based on the torque, this motor meets the desired 15.21 oz·in with any power supply.

The rotor intertia, per the motor specification is 0.28 oz·in^2 or 0.0000454 lb·in·sec^2.

Load/Motor inertia ratio:

$J_{(screw\ +\ load)} \div J_{motor}$ = 0.001362 lb·in·sec^2 ÷ 0.0000454 lb·in·sec^2

= 30

The ratio of 30 is well above the desired ratio of 10, so the STP-MTR-17040 motor is not suitable.

Consider STP-MTR-17048:

STP-MTR-17048 Torque vs Speed (1.8° step motor; 1/2 stepping)

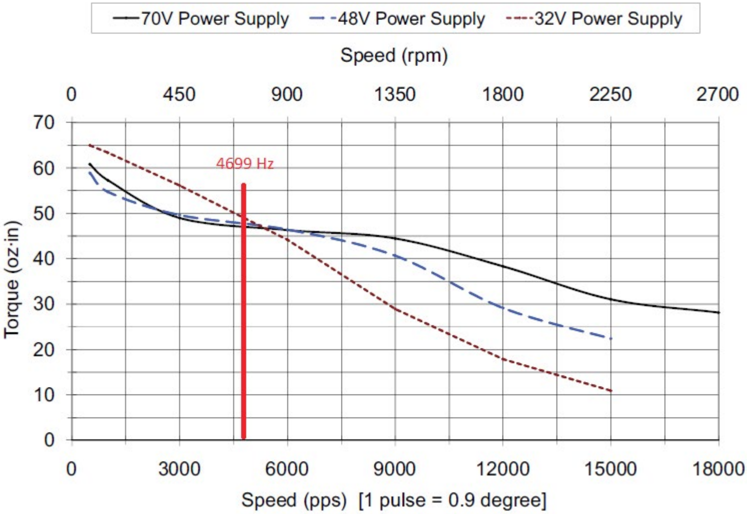


Figure 3: Torque for STP-MTR-17048 at 4.7 kHz

For the purpose of this example, the torque vs. speed curves for this motor at 4.7 kHz will be approximately 48 oz·in at 48VDC, and 46 oz·in at 70VDC. This motor also meets the desired 15.21 oz·in requirement.

The rotor intertia, per the motor specification is 0.45 oz·in^2 or 0.000024 lb·in·sec^2.

Load/Motor inertia ratio:

$\mathbf{J_{(screw\ +\ load)} \div J_{motor} =}$ 0.001362 lb·in·sec^2 ÷ 0.000024 lb·in·sec^2

= 18.683

The ratio of 18.683 is still above the desired ratio of 10, so the STP-MTR-17048 motor is not suitable.

We can keep increasing the motor size, or maybe change the reflected load inertia by changing the reduction ratio from 2:1 to 3:1.

Reduction Ratio 3:1 (Step 2 revisited) – Determine the Positioning Resolution of the Load

One revolution of the lead screw shaft advances 0.6 inches. We are looking for a 0.001 inch per step precision.

Check to see if 400 pulses/rev will achieve the desired precision:

With a reduction of 3:1, there will be three motor shaft revolutions to get a displacement of 0.6 inches.

(3 rev) x (400 pulses/rev) = 1200 pulses for every 0.6 inches of displacement.

Therefore, (0.6 in) / (1200 pulses) = 0.0005 in/pulse.

This is within the desired 0.001 in/step.

How many pulses are needed for the displacement?

With the 3:1 gear reduction, the stepping system can be set at 400 steps/rev to exceed the required load positioning resolution.

Since the lead screw advances 0.6 inches / rev, in the required stroke of 4.5 inches we will need:
(4.5 in) / (0.6 rev/in) = 7.5 revolutions on the lead screw

Since we have a reduction of 3:1, the motor shaft shall rotate 22.5 revolutions.

Reduction Ratio 3:1 (Step 3 revisited) – Determine the Motion Profile

Since we know that 400 pulses gets one revolution on the motor and we need 22.5 revolutions, then 400 x 22.5 = 9,000 pulses to move 4.5 inches.

From **Equation** ④, the indexing frequency for a trapezoidal move is:

$$\mathbf{f_{TRAP} = (P_{total} - (f_{start} \times t_{ramp})) \div (t_{total} - t_{ramp})}$$
$$= (9{,}000 - (20 \times 0.425s)) \div (1.7 - 0.425) = 8{,}991.5 \text{ Hz},$$

where the starting speed is 20Hz

(8,991.5 Hz) / (400 steps/rev) = 22.48 rev/s

To get it in rpm, (22.48 rev/s) x (60 s/min) = 1349 rpm.

Reduction Ratio 3:1 (Step 4 revisited) – Determine the Required Motor Torque

Using the equations in **Table 1**:
(Total inertia seen by the motor is the sum of all inertias)

$$J_{total} = J_{motor} + J_{gear} + ((J_{coupling} + J_{screw} + J_W) \div \mathbf{i^2})$$

For this example, we assume the gearbox and coupling inertias are zero.

Load inertia:

$$\mathbf{J_W = (W \div (g \times e)) \times (1 \div 2\pi P)^2}$$
$$= (200 \div (386 \times 0.9)) \times (1 \div 2 \times 3.1416 \times 1.67)^2$$
$$= 0.00523 \text{ lb·in·sec}^2$$

Lead screw inertia:

$$\mathbf{J_{screw} = (\pi \times L \times \rho \times r^4) \div (2g)}$$
$$= (3.1416 \times 23.6 \times 0.28 \times 0.3^4) \div (2 \times 386)$$
$$= 0.0002178 \text{ lb·in·sec}^2$$

The inertia of the load and screw reflected to the motor is the sum of both values divided by the square of the reduction ratio

$$\mathbf{J_{(screw\ +\ load)\ referred\ to\ motor} = ((J_{screw} + J_W) \div i^2)}$$

$$J_{total\ less\ the\ motor\ inertia} = ((0.0002178 + 0.00523) \div 3^2) = 0.000605 \text{ lb·in·sec}^2$$

The dynamic torque required to accelerate the inertia (without the motor rotor inertia) is:

$$\mathbf{T_{accel} = J_{total} \times ((\Delta_{rpm} \div \Delta_{time}) \times (2\pi \div 60))}$$
$$= 0.000605 \times ((1349 \div 0.2125) \times (2 \times 3.1416 \div 60))$$
$$= 0.4022 \text{ lb·in}$$

Determine the running torque, or in this case the friction torque:

The forces are:

$F_{total} = F_{ext} + F_{friction} + F_{gravity}$ [External forces and gravity are zero in this case.]

$= 0 + (\mu \times W \times \cos\theta) + 0$

$= 0.05 \times 200 = 10$ lb

And the formula to be used is:

$T_{run} = ((F_{total} \div (2\pi P)) + T_{preload}) \div i$

$= (10 \div (2 \times 3.1416 \times 1.67)) \div 3$ [related to the motor side]

$= 0.3177$ lb·in [where, we have assumed the preload torque to be zero]

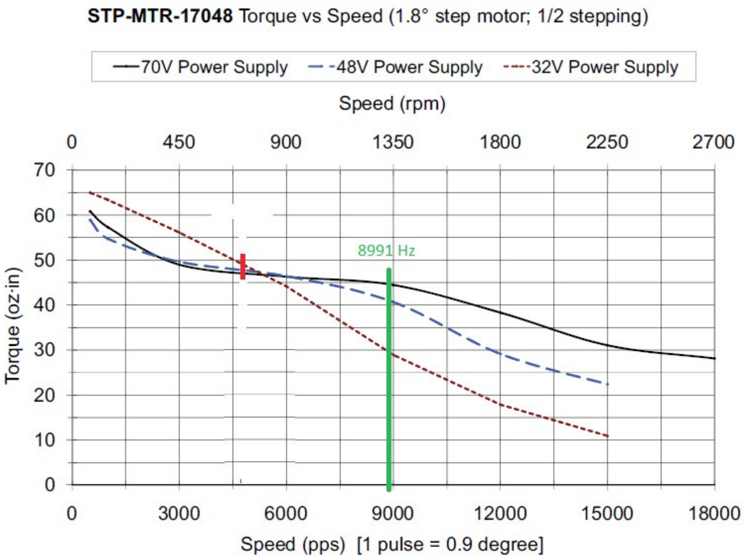
From **Equation** ⑤, the minimum required motor torque @ 1349 rpm is:

$T_{motor} = T_{accel} + T_{run}$

$= (0.4022$ lb·in$) + (0.3177$ lb·in$)$

$= 0.7199$ lb·in, or 11.51 oz·in

Consider STP-MTR-17048 with the new values:



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kHz will be approximately 26 oz·in at 32VDC and 44 oz·in at 70VDC. Based on the torque, this motor meets the desired 11.51 oz·in requirement.

The rotor intertia, per the motor specification is 0.45 oz·in^2 or 0.000024 lb·in·sec^2.

Load/Motor inertia ratio:

$$\mathbf{J_{(screw\ +\ load)} \div J_{motor}} = 0.00605\ \text{lb·in·sec}^2 \div 0.000024\ \text{lb·in·sec}^2$$

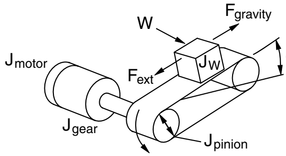
$$= \mathbf{2.52}$$

The ratio of 2.52 is below the desired ratio of 10, so the STP-MTR-17048 motor is suitable.

A small change in the mechanical design allowed us to use the motor STP-MTR-17048. The torque in the point of operation is more than enough, and the ratio of intertia criteria is fulfilled with any level of voltage in the drive. To be sure the safety factor is high, it would be better to select a 48V power supply and drive.

Belt Drive – Example Calculations

Step 1 – Define the Actuator and Motion Requirements



Weight of table and workpiece = 3 lb
External force = 0 lb
Friction coefficient of sliding surfaces = 0.05
Angle of table = 0º
Belt and pulley efficiency = 0.8
Pulley diameter = 1.5 inch
Pulley thickness = 0.75 inch
Pulley material = aluminum
Desired Resolution = 0.001 inch/step
Gear Reducer = 5:1
Stroke = 50 inch
Move time = 4.0 seconds
Accel and decel time = 1.0 seconds

Definitions
d_{load} = lead or distance the load moves per revolution of the actuator's drive shaft (P = pitch = $1/d_{load}$)
D_{total} = total move distance
θ_{step} = driver step resolution (steps/rev_{motor})
i = gear reduction ratio (rev_{motor}/$rev_{gearshaft}$)
T_{accel} = motor torque required to accelerate and decelerate the total system inertia (including motor inertia)
T_{run} = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.
t_{total} = move time

Step 2 – Determine the Positioning Resolution of the Load

Rearranging **Equation** ④ to calculate the required stepping drive resolution:

$\theta_{step} = (\mathbf{d_{load}} \div \mathbf{i}) \div \mathbf{L_\theta}$
$= ((3.14 \times 1.5) \div 5) \div 0.001$
$= 942$ steps/rev
where $\mathbf{d_{load}} = \pi \times$ Pulley Diameter.

With the 5:1 gear reduction, the stepping system can be set at 1000 steps/rev to slightly exceed the required load positioning resolution.

Reduction is almost always required with a belt drive, and a 5:1 planetary gearhead is common.

Step 3 – Determine the Motion Profile

From **Equation** ①, the total pulses to make the required move is:

$\mathbf{P_{total}} = (\mathbf{D_{total}} \div (\mathbf{d_{load}} \div \mathbf{i})) \times \theta_{step}$
$= 50 \div ((3.14 \times 1.5) \div 5) \times 1000$
$\approx 53{,}079$ pulses

From **Equation** ④, the running frequency for a trapezoidal move is:

$\mathbf{f_{TRAP}} = (\mathbf{P_{total}} - (\mathbf{f_{start}} \times \mathbf{t_{ramp}})) \div (\mathbf{t_{total}} - \mathbf{t_{ramp}})$
$= 53{,}079 \div (4 - 1)$
$\approx 17{,}693$ Hz
where accel time is 25% of total move time and starting speed is zero.
$= 17{,}693$ Hz x (60 sec/1 min) ÷ 1000 steps/rev
$\approx 1{,}062$ RPM motor speed

Step 4 – Determine the Required Motor Torque

Using the equations in **Table 1**:

$\mathbf{J_{total}} = \mathbf{J_{motor}} + \mathbf{J_{gear}} + ((\mathbf{J_{pulleys}} + \mathbf{J_W}) \div \mathbf{i}^2)$

For this example, let's assume the gearbox inertia is zero.

$\mathbf{J_W} = (\mathbf{W} \div (g \times e)) \times r^2$
$= (3 \div (386 \times 0.8)) \times 0.75^2$
≈ 0.0055 lb·in·sec²

Pulley inertia (remember there are two pulleys) can be calculated as:

$\mathbf{J_{pulleys}} \approx ((\pi \times L \times \rho \times r^4) \div (2g)) \times 2$

$\approx ((3.14 \times 0.75 \times 0.098 \times 0.754) \div (2 \times 386)) \times 2$
≈ 0.00019 lb·in·sec²

The inertia of the load and pulleys reflected to the motor is:

$\mathbf{J}_{\mathbf{(pulleys + load) to motor}} = ((\mathbf{J}_{\mathbf{pulleys}} + \mathbf{J}_{\mathbf{W}}) \div \mathbf{i}^2)$
$\approx ((0.0055 + 0.00019) \div 52) \approx 0.00023$ lb·in·sec²

The torque required to accelerate the inertia is:

$\mathbf{T}_{\mathbf{acc}} \approx \mathbf{J}_{\mathbf{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$
$= 0.00023 \times (1062 \div 1) \times 0.1$
$= 0.025$ lb·in

$\mathbf{T}_{\mathbf{run}} = (\mathbf{F}_{\mathbf{total}} \times \mathbf{r}) \div \mathbf{i}$

$\mathbf{F}_{\mathbf{total}} = \mathbf{F}_{\mathbf{ext}} + \mathbf{F}_{\mathbf{friction}} + \mathbf{F}_{\mathbf{gravity}}$
$= 0 + \mu W\cos\theta + 0 = 0.05 \times 3 = 0.15$ lb

$\mathbf{T}_{\mathbf{run}} = (0.15 \times 0.75) \div 5$
≈ 0.0225 lb·in

From **Equation** ⑤, the required motor torque is:

$\mathbf{T}_{\mathbf{motor}} = \mathbf{T}_{\mathbf{accel}} + \mathbf{T}_{\mathbf{run}} = 0.025 + 0.0225 \approx 0.05$ lb·in

However, this is the required motor torque before we have picked a motor and included the motor inertia.

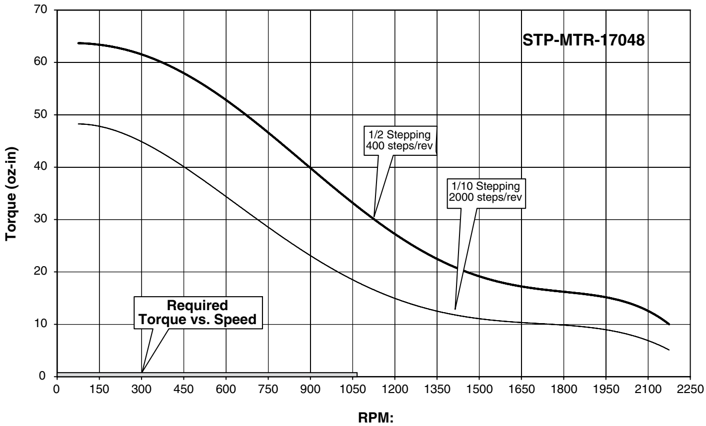
Step 5 – Select & Confirm the Stepping Motor & Driver System

It looks like a reasonable choice for a motor would be the STP-MTR-17048 or NEMA 17 motor. This motor has an inertia of:

$\mathbf{J}_{\mathbf{motor}} = 0.00006$ lb·in·sec²

The actual motor torque would be modified:

$\mathbf{T}_{\mathbf{accel}} = \mathbf{J}_{\mathbf{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$
$= (0.00023 + \mathbf{0.00006}) \times (1062 \div 1) \times 0.1 \approx 0.03$ lb·in
so that:
$\mathbf{T}_{\mathbf{motor}} = \mathbf{T}_{\mathbf{accel}} + \mathbf{T}_{\mathbf{run}}$
$= 0.03 + 0.0225 \approx 0.0525$ lb·in ≈ 0.84 oz·in



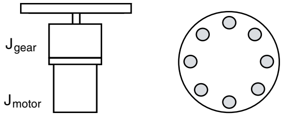
It looks like the STP-MTR-17048 stepping motor will work. However, we still need to check the load to motor inertia ratio:

Ratio = $\mathbf{J_{(pulleys\ +\ load)\ to\ motor}} \div \mathbf{J_{motor}}$
$= 0.00023 \div 0.00006 = 3.8$

It is best to keep the load-to-motor inertia ratio below 10, so 3.8 is within an acceptable range.

Index Table – Example Calculations

Step 1 – Define the Actuator and Motion Requirements



Diameter of index table = 12 inch
Thickness of index table = 2 inch
Table material = steel
Number of workpieces = 8
Desired Resolution = 0.036º
Gear Reducer = 25:1
Index angle = 45º
Index time = 0.7 seconds

Definitions
d_{load} = lead or distance the load moves per revolution of the actuator's drive shaft (P = pitch = $1/d_{load}$)
D_{total} = total move distance
θ_{step} = driver step resolution (steps/rev_{motor})
i = gear reduction ratio (rev_{motor}/$rev_{gearshaft}$)
T_{accel} = motor torque required to accelerate and decelerate the total system inertia (including motor inertia)
T_{run} = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.
t_{total} = move time

Step 2 – Determine the Positioning Resolution of the Load

Rearranging **Equation** ④ to calculate the required stepping drive resolution:

$$\theta_{step} = (\mathbf{d_{load}} \div \mathbf{i}) \div \mathbf{L_\theta}$$
$$= (360^\circ \div 25) \div 0.036^\circ$$
$$= 400 \text{ steps/rev}$$

With the 25:1 gear reduction, the stepping system can be set at 400 steps/rev to equal the required load positioning resolution.

It is almost always necessary to use significant gear reduction when controlling a large inertia disk.

Step 3 – Determine the Motion Profile

From **Equation** ①, the total pulses to make the required move is:

$$\mathbf{P}_{\mathbf{total}} = (\mathbf{D}_{\mathbf{total}} \div (\mathbf{d}_{\mathbf{load}} \div \mathbf{i})) \times \theta_{\mathbf{step}}$$
$$= (45^{o} \div (360^{o} \div 25) \times 400$$
$$= 1250 \text{ pulses}$$

From **Equation** ④, the running frequency for a trapezoidal move is:

$$\mathbf{f}_{\mathbf{TRAP}} = (\mathbf{P}_{\mathbf{total}} - (\mathbf{f}_{\mathbf{start}} \times \mathbf{t}_{\mathbf{ramp}})) \div (\mathbf{t}_{\mathbf{total}} - \mathbf{t}_{\mathbf{ramp}})$$
$$= 1{,}250 \div (0.7 - 0.17) \approx 2{,}360 \text{ Hz}$$

where accel time is 25% of total move time and starting speed is zero.

$$= 2{,}360 \text{ Hz} \times (60 \text{ sec}/1 \text{ min}) \div 400 \text{ steps/rev}$$
$$\approx 354 \text{ RPM}$$

Step 4 – Determine the Required Motor Torque

Using the equations in **Table 1**:

$$\mathbf{J}_{\mathbf{total}} = \mathbf{J}_{\mathbf{motor}} + \mathbf{J}_{\mathbf{gear}} + (\mathbf{J}_{\mathbf{table}} \div \mathbf{i}^2)$$

For this example, let's assume the gearbox inertia is zero.

$$\mathbf{J}_{\mathbf{table}} \approx (\pi \times L \times \rho \times r^4) \div (2g)$$
$$\approx (3.14 \times 2 \times 0.28 \times 1296) \div (2 \times 386)$$
$$\approx 2.95 \text{ lb·in·sec}^2$$

The inertia of the indexing table reflected to the motor is:

$$\mathbf{J}_{\mathbf{table\ to\ motor}} = \mathbf{J}_{\mathbf{table}} \div \mathbf{i}^2$$
$$\approx 0.0047 \text{ lb·in·sec}^2$$

The torque required to accelerate the inertia is:

$$\mathbf{T}_{\mathbf{accel}} \approx \mathbf{J}_{\mathbf{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$$
$$= 0.0047 \times (354 \div 0.17) \times 0.1$$
$$\approx 1.0 \text{ lb·in}$$

From **Equation** ⑤, the required motor torque is:

$$\mathbf{T}_{\mathbf{motor}} = \mathbf{T}_{\mathbf{accel}} + \mathbf{T}_{\mathbf{run}}$$
$$= 1.0 + 0 = 1.0 \text{ lb·in}$$

However, this is the required motor torque before we have picked a motor and included the motor inertia.

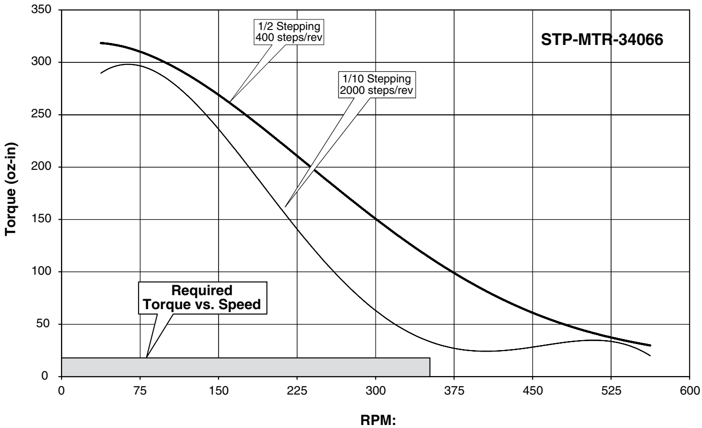
Step 5 – Select & Confirm the Stepping Motor & Driver System

It looks like a reasonable choice for a motor would be the STP-MTR-34066, or NEMA 34 motor. This motor has an inertia of:

$\mathbf{J_{motor}} = 0.0012$ lb·in·sec²

The actual motor torque would be modified:

$\mathbf{T_{accel}} = \mathbf{J_{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$
$= (0.0047 + \mathbf{0.0012}) \times (354 \div 0.17) \times 0.1$
≈ 1.22 lb·in
so that:
$\mathbf{T_{motor}} = \mathbf{T_{accel}} + \mathbf{T_{run}}$
$= 1.22 + 0$
$= 1.22$ lb·in $= 19.52$ oz·in



It looks like the STP-MTR-34066 stepping motor will work. However, we still need to check the load to motor inertia ratio:

$\mathbf{Ratio} = \mathbf{J_{table\ to\ motor}} \div \mathbf{J_{motor}}$
$= 0.0047 \div 0.0012 = 3.9$

It is best to keep the load-to-motor inertia ratio below 10, so 3.9 is within an acceptable range.

Engineering Unit Conversion Tables, Formulae, & Definitions:

Conversion of Length							
To convert A to B, multiply A by the entry in the table.		B					
		µm	mm	m	mil	in	ft
A	µm	1	1.000E–03	1.000E–06	3.937E–02	3.937E–05	3.281E–06
	mm	1.000E+03	1	1.000E–03	3.937E+01	3.937E–02	3.281E–03
	m	1.000E+06	1.000E+03	1	3.937E+04	3.937E+01	3.281E+00
	mil	2.540E+01	2.540E–02	2.540E–05	1	1.000E–03	8.330E–05
	in	2.540E+04	2.540E+01	2.540E–02	1.000E+03	1	8.330E–02
	ft	3.048E+05	3.048E+02	3.048E–01	1.200E+04	1.200E+01	1

Conversion of Torque							
To convert A to B, multiply A by the entry in the table.		B					
		N·m	kg·m	kg·cm	oz·in	lb·in	lb·ft
A	N·m	1	1.020E–01	1.020E+01	1.416E+02	8.850E+00	7.380E-01
	kg·m	9.810E+00	1	1.000E+02	1.390E+03	8.680E+01	7.230E+00
	kg·cm		1.000E–02	1	1.390E+01	8.680E–01	7.230E–02
	oz·in	7.060E–03	7.200E–04	7.200E–02	1	6.250E–02	5.200E–03
	lb·in	1.130E–01	1.150E–02	1.150E+00	1.600E+01	1	8.330E–02
	lb·ft	1.356E+00	1.380E–01	1.383E+01	1.920E+02	1.200E+01	1

Conversion of Moment of Inertia								
To convert A to B, multiply A by the entry in the table.		B						
		kg·m²	kg·cm·s²	oz·in·s²	lb·in·s²	oz·in²	lb·in²	lb·ft²
A	kg·m²	1	1.020E+01	1.416E+02	8.850E+00	5.470E+04	3.420E+03	2.373E+01
	kg·cm·s²	9.800E–02	1	1.388E+01	8.680E–01	5.360E+03	3.350+02	2.320E+00
	oz·in·s²	7.060E–03	7.190E–02	1	6.250E–02	3.861E+02	2.413E+01	1.676E–01
	lb·in·s²	1.130E–01	1.152E+00	1.600E+01	1	6.180E+03	3.861E+02	2.681E+00
	oz·in²	1.830E–05	1.870E–04	2.590E–03	1.620E–04	1	6.250E–02	4.340E–04
	lb·in²	2.930E–04	2.985E–03	4.140E–02	2.590E–03	1.600E+01	1	6.940E–03
	lb·ft²	4.210E–02	4.290E–01	5.968E+00	3.730E–01	2.304E+03	1.440E+02	1

Engineering Unit Conversion Tables, Formulae, & Definitions (cont'd):

General Formulae & Definitions	
Description:	**Equations:**
Gravity	gravity = 9.8 m/s^2; 386 in/s^2
Torque	$T = J \cdot \alpha$; $\alpha = rad/s^2$
Power (Watts)	P (W) = T (N·m) · ω (rad/s)
Power (Horsepower)	P (hp) = T (lb·in) · ν (rpm) / 63,024
Horsepower	1 hp = 746W
Revolutions	1 rev = 1,296,000 arc·sec / 21,600 arc·min

Equations for Straight-Line Velocity & Constant Acceleration	
Description:	**Equations:**
Final velocity	$v_f = v_i + at$ final velocity = (initial velocity) + (acceleration)(time)
Final position	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$ final position = initial position + [(1/2)(initial velocity + final velocity)(time)]
Final position	$x_f = x_i + v_i t + \frac{1}{2}at^2$ final position = initial position + (initial velocity)(time) + (1/2)(acceleration)(time squared)
Final velocity squared	$v_f^2 = v_i^2 + 2a(x_f - x_i)$ final velocity squared = initial velocity squared + [(2)(acceleration)(final position – initial position)]

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