

# SELECTING THE SURESERVO® SYSTEM



## APPENDIX

# A

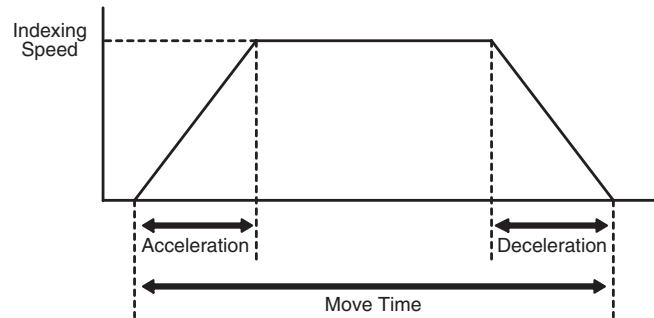
### CONTENTS OF THIS CHAPTER...

<i>Selecting the SureServo® Servo System. . . . .</i>	<i>A-2</i>
<i>The Selection Procedure . . . . .</i>	<i>A-2</i>
<i>How many pulses from the PLC to make the move? . . . . .</i>	<i>A-2</i>
<i>What is the positioning resolution of the load? . . . . .</i>	<i>A-3</i>
<i>What is the indexing speed to accomplish the move time? . . . . .</i>	<i>A-3</i>
<i>Calculating the Required Torque . . . . .</i>	<i>A-4</i>
<i>Leadscrew – Example Calculations . . . . .</i>	<i>A-8</i>
<i>Step 1 – Define the Actuator and Motion Requirements. . . . .</i>	<i>A-8</i>
<i>Step 2 – Determine the Positioning Resolution of the Load . . . . .</i>	<i>A-8</i>
<i>Step 3 – Determine the Motion Profile. . . . .</i>	<i>A-9</i>
<i>Step 4 – Determine the Required Motor Torque . . . . .</i>	<i>A-9</i>
<i>Step 5 – Select and Confirm the Servo Motor and Driver System . . . . .</i>	<i>A-10</i>
<i>Belt Drive – Example Calculations . . . . .</i>	<i>A-11</i>
<i>Step 1 – Define the Actuator and Motion Requirements. . . . .</i>	<i>A-11</i>
<i>Step 2 – Determine the Positioning Resolution of the Load . . . . .</i>	<i>A-11</i>
<i>Step 3 – Determine the Motion Profile. . . . .</i>	<i>A-12</i>
<i>Step 4 – Determine the Required Motor Torque . . . . .</i>	<i>A-12</i>
<i>Step 5 – Select and Confirm the Servo Motor and Driver System . . . . .</i>	<i>A-13</i>
<i>Index Table – Example Calculations . . . . .</i>	<i>A-14</i>
<i>Step 1 – Define the Actuator and Motion Requirements. . . . .</i>	<i>A-14</i>
<i>Step 2 – Determine the Positioning Resolution of the Load . . . . .</i>	<i>A-14</i>
<i>Step 3 – Determine the Motion Profile. . . . .</i>	<i>A-14</i>
<i>Step 4 – Determine the Required Motor Torque . . . . .</i>	<i>A-15</i>
<i>Step 5 – Select and Confirm the Servo Motor and Driver System . . . . .</i>	<i>A-15</i>
<i>Engineering Unit Conversion Tables, Formulas, &amp; Definitions. . . . .</i>	<i>A-16</i>

## SELECTING THE SURESERVO® SERVO SYSTEM

The selection of your SureServo® servo system follows a defined process. Let's go through the process and define some useful relationships and equations. We will use this information to work some typical examples along the way.

### THE SELECTION PROCEDURE



The motor provides for the required motion of the load through the actuator (mechanics that are between the motor shaft and the load or workpiece). Key information to accomplish the required motion is:

- total number of pulses from the PLC
- positioning resolution of the load
- indexing speed (or PLC pulse frequency) to achieve the move time
- required motor torque (including the 25% safety factor)
- load to motor inertia ratio

In the final analysis, we need to achieve the required motion with acceptable positioning accuracy.

### HOW MANY PULSES FROM THE PLC TO MAKE THE MOVE?

The total number of pulses to make the entire move is expressed with the equation:

**Equation 1:**  $P_{total} = \text{total pulses} = (D_{total} \div (d_{load} \div i)) \times \theta_{count}$

- $D_{total}$  = total move distance
- $d_{load}$  = lead or distance the load moves per revolution of the actuator's drive shaft ( $P = \text{pitch} = 1/d_{load}$ )
- $\theta_{count}$  = servo resolution (counts/rev<sub>motor</sub>) (default = 10,000)
- $i$  = gear reduction ratio (rev<sub>motor</sub>/rev<sub>gearshaft</sub>)

**Example 1:** The motor is directly attached to a disk and we need to move the disk 5.5 revolutions. How many pulses does the PLC need to send to the driver?

$$P_{total} = (5.5 \text{ rev}_{disk} \div (1 \text{ rev}_{disk}/\text{rev}_{driveshaft} \div 1 \text{ rev}_{motor}/\text{rev}_{driveshaft})) \\ 5.5 \div (1.0 \div 10) \times 10,000 = 550,000 \\ \times 10,000 \text{ counts/rev}_{motor} \\ = 55,000 \text{ pulses}$$

**Example 2:** The motor is directly attached to a ballscrew where one turn of the ballscrew results in 20 mm of linear motion and we need to move 45 mm. How many pulses do we need to send the driver?

$$P_{total} = (45 \text{ mm} \div (20 \text{ mm}/\text{rev}_{screw} \div 1 \text{ rev}_{motor}/\text{rev}_{screw})) \times 10,000 \text{ counts/rev}_{motor} \\ = 22,500 \text{ pulses}$$

45 mm	1 rev <sub>screw</sub>	1 rev <sub>motor</sub>	10,000 pulses
move	20 mm	1 rev <sub>screw</sub>	1 rev <sub>motor</sub>

**Example 3:** Let's add a 2:1 belt reduction between the motor and ballscrew in example 2. Now how many pulses do we need to make the 45 mm move?

$$P_{total} = (45 \text{ mm} \div (20\text{mm}/\text{rev}_{screw} \div 2 \text{ rev}_{motor}/\text{rev}_{screw})) \times 10,000 \text{ counts/rev}_{motor} \\ = 45,000 \text{ pulses}$$

**WHAT IS THE POSITIONING RESOLUTION OF THE LOAD?**

We want to know how far the load will move for one command pulse. The equation to determine the positioning resolution is:

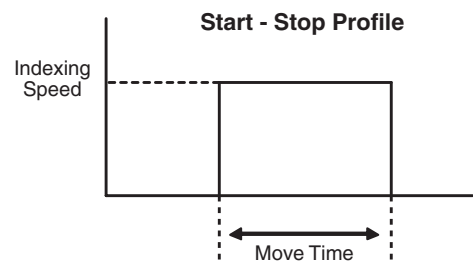
**Equation 2:**  $L_{\theta} = \text{load positioning resolution} = (d_{\text{load}} \div i) \div \theta_{\text{count}}$

**Example 4:** What is the positioning resolution for the system in example 3?

- $L_{\theta} = (d_{\text{load}} \div i) \div \theta_{\text{count}}$   
 $= (20 \text{ mm/rev}_{\text{screw}} \div 2 \text{ rev}_{\text{motor}}/\text{rev}_{\text{screw}}) \div 10,000 \text{ counts/rev}_{\text{motor}}$   
 $= 0.001 \text{ mm/count}$   
 $\approx 0.00004 \text{ in/count}$

**WHAT IS THE INDEXING SPEED TO ACCOMPLISH THE MOVE TIME?**

The most basic type of motion profile is a “start-stop” profile where there is no acceleration or deceleration period. This type of motion profile is only used for low speed applications because the load is “jerked” from one speed to another and the servo system may experience a position deviation error if excessive speed changes are attempted. The equation to find indexing speed for “start-stop” motion is:



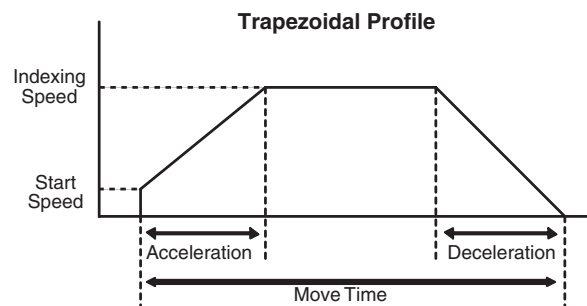
**Equation 3:**  $f_{SS} = \text{indexing speed for Start/Stop profiles} = P_{\text{total}} \div t_{\text{total}}$

- $t_{\text{total}} = \text{move time}$

**Example 5:** What is the indexing speed to make a “start-stop” move with 10,000 pulses in 800 ms?

- $f_{SS} = \text{indexing speed} = P_{\text{total}} \div t_{\text{total}} = 10,000 \text{ pulses} \div 0.8 \text{ seconds}$   
 $= 12,500 \text{ Hz.}$

For higher speed operation, the “trapezoidal” motion profile includes controlled acceleration & deceleration and, in some cases, an initial non-zero starting speed. With the acceleration and deceleration periods equally set, the indexing speed can be found using the equation:



**Equation 4:**  $f_{TRAP} = (P_{\text{total}} - (f_{\text{start}} \times t_{\text{ramp}})) \div (t_{\text{total}} - t_{\text{ramp}})$   
 for trapezoidal motion profiles

- $f_{\text{start}} = \text{starting speed}$
- $t_{\text{ramp}} = \text{acceleration or deceleration time}$

**Example 6:** What is the required indexing speed to make a “trapezoidal” move in 1.8s, accel/decel time of 200 ms each, 100,000 total pulses, and a starting speed of 40 Hz?

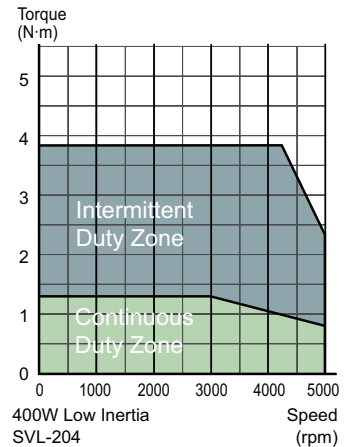
- $f_{TRAP} = (100,000 \text{ pulses} - (40 \text{ pulses/sec} \times 0.2 \text{ sec})) \div (1.8 \text{ sec} - 0.2 \text{ sec})$   
 $\approx 62,375 \text{ Hz}$

**CALCULATING THE REQUIRED TORQUE**

The required torque is the sum of acceleration (or deceleration) torque and the running torque. The equation for required motor torque is:

**Equation 5:**  $T_{motor} = T_{accel (or decel)} + T_{run}$

- $T_{accel}$  = motor torque required to accelerate the total system inertia (including motor inertia).
- $T_{decel}$  = motor torque required to decelerate; not always the same as acceleration.
- $T_{run}$  = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.
- Continuous Duty Zone means the system can provide the torque under the curve 100% of the time.
- Intermittent Duty Zone means the system can provide the torque under the curve LESS THAN 100% of the time.



The amount of time the system can operate in this region depends on the amount of torque. In general, the higher the torque, the shorter period of time is allowed. See overload curves information in Chapter 1. If a system requires more than rated torque occasionally, but only for a short time, the system can do it. Running in this zone continuously will result in an overload fault.

In **Table 1** we show how to calculate torque required to accelerate or decelerate an inertia from one speed to another and the calculation of running torque for common mechanical actuators.

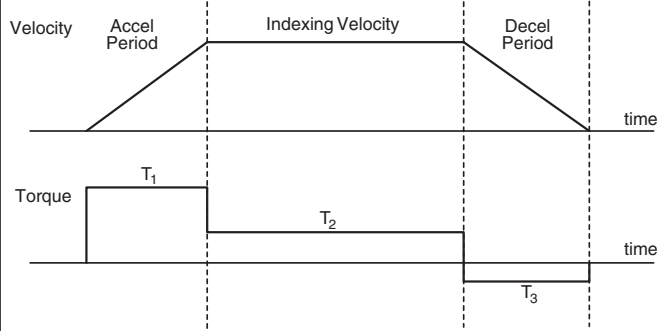
**Table 1 – Calculate the Torque for “Acceleration” and “Running”**

The torque required to accelerate or decelerate a constant inertia with a linear change in velocity is:

**Equation 6:**

$$T_{\text{accel}} = J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times (2\pi \div 60)$$

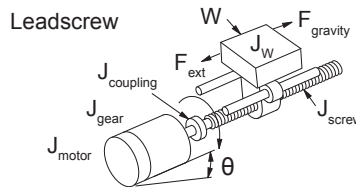
- $J_{\text{total}}$  is the motor inertia plus load inertia (“reflected” to the motor shaft).
- The  $(2\pi \div 60)$  is a factor used to convert “change in speed” expressed in rpm into angular speed (radians/second).
- Refer to information in this table to calculate “reflected” load inertia for several common shapes and mechanical mechanisms.



**Example 7:** What is the required torque to accelerate an inertia of 0.002 lb·in·sec<sup>2</sup> (motor inertia is 0.0004 lb·in·sec<sup>2</sup> and “reflected” load inertia is 0.0016 lb·in·sec<sup>2</sup>) from zero to 600 rpm in 50 ms?

- $T_{\text{accel}} = 0.002 \text{ lb}\cdot\text{in}\cdot\text{sec}^2 \times (600 \text{ rpm} \div 0.05 \text{ seconds}) \times (2\pi \div 60)$   
 $\approx 2.5 \text{ lb}\cdot\text{in}$

**Leadscrew Equations**



Description:	Equations:	
<b>Definitions</b>	coefficient of friction = $\mu$ efficiency = $e$ gear ratio = $i$ incline angle = $\theta$ pitch = $P$	$\mu$ = coefficient of friction $e$ = efficiency $i$ = gear ratio $\theta$ = incline angle $P$ = pitch
<b>Motor rpm</b>	$n_{\text{motor}} = (v_{\text{load}} \times P) \times i$ ; $n_{\text{motor}}$ (rpm); $v_{\text{load}}$ (in/min)	
<b>Torque required to accelerate and decelerate the load</b>	$T_{\text{accel}} = J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$	
<b>Motor total inertia</b>	$J_{\text{total}} = J_{\text{motor}} + J_{\text{gear}} + ((J_{\text{coupling}} + J_{\text{screw}} + J_{\text{nut}}) \div i^2)$	
<b>Inertia of the load</b>	$J_{\text{W}} = (W \div (g \times e)) \times (1 \div 2 \pi P)^2$	
<b>Pitch</b>	$P = \text{revs/inch of travel}$	
<b>Running torque</b>	$T_{\text{run}} = ((F_{\text{total}} \div (2 \pi P)) + T_{\text{preload}}) \div i$	
<b>Torque due to preload on the ballscrew</b>	$T_{\text{preload}} = \text{ballscrew nut preload to minimize backlash}$	
<b>Force total</b>	$F_{\text{total}} = F_{\text{ext}} + F_{\text{friction}} + F_{\text{gravity}}$	
<b>Force of friction</b>	$F_{\text{friction}} = \mu W \cos\theta$	
<b>Force of gravity</b>	$F_{\text{gravity}} = W \sin\theta$	
Typical Leadscrew Data		
<b>Material:</b>	<b><math>e</math> = efficiency</b>	<b>Material:</b>
<b>ball nut</b>	0.90	<b>steel on steel</b>
<b>acme with plastic nut</b>	0.65	<b>steel on steel (lubricated)</b>
<b>acme with metal nut</b>	0.40	<b>teflon on steel</b>
		<b>ball bushing</b>
		<b><math>\mu</math> = coef. of friction</b>
		0.580
		0.150
		0.040
		0.003

(table continued next page)

Table 1 – Calculate the Torque for “Acceleration” and “Running” (continued from previous page)

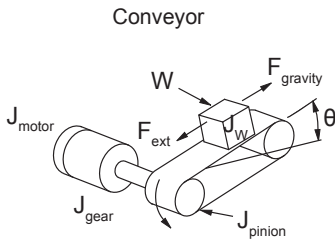
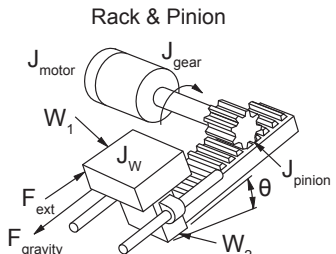
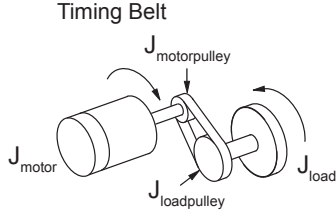
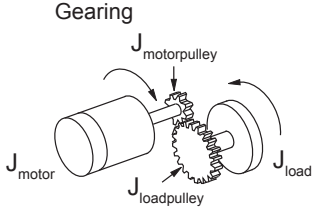
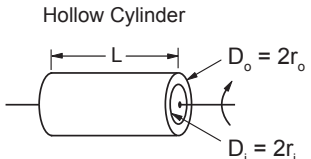
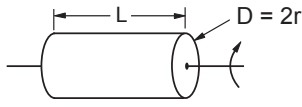
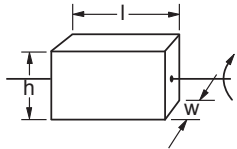
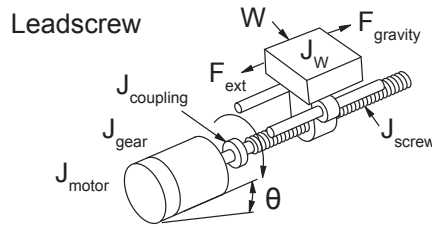
<b>Belt Drive (or Rack &amp; Pinion) Equations</b>	
 <p>Conveyor</p>	 <p>Rack &amp; Pinion</p>
<b>Description:</b>	<b>Equations:</b> [ i = gear ratio ]
<b>Motor rpm</b>	$n_{\text{motor}} = (v_{\text{load}} \times 2 \pi r) \times i$
<b>Torque required to accelerate and decelerate the load</b>	$T_{\text{accel}} = J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$
<b>Inertia of the load</b>	$J_{\text{total}} = J_{\text{motor}} + J_{\text{gear}} + ((J_{\text{pinion}} + J_{\text{W}}) \div i^2)$
<b>Inertia of the load</b>	$J_{\text{W}} = (W \div (g \times e)) \times r^2$ ; $J_{\text{W}} = ((W_1 + W_2) \div (g \times e)) \times r^2$
<b>Radius of pulleys</b>	$r = \text{radius of pinion or pulleys (inch)}$
<b>Running torque</b>	$T_{\text{run}} = (F_{\text{total}} \times r) \div i$
<b>Force total</b>	$F_{\text{total}} = F_{\text{ext}} + F_{\text{friction}} + F_{\text{gravity}}$
<b>Force of gravity and Force of friction</b>	$F_{\text{gravity}} = W \sin \theta$ ; $F_{\text{friction}} = \mu W \cos \theta$
<b>Belt (or Gear) Reducer Equations</b>	
 <p>Timing Belt</p>	 <p>Gearing</p>
<b>Description:</b>	<b>Equations:</b> [ i = gear ratio ]
<b>Motor rpm</b>	$n_{\text{motor}} = n_{\text{load}} \times i$
<b>Torque required to accelerate and decelerate the load</b>	$T_{\text{accel}} = J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$
<b>Inertia of the load</b>	$J_{\text{total}} = J_{\text{motor}} + J_{\text{motorpulley}} + ((J_{\text{loadpulley}} + J_{\text{load}}) \div i^2)$
<b>Motor torque</b>	$T_{\text{motor}} \times i = T_{\text{load}}$
<b>Inertia of Hollow Cylinder Equations</b>	
 <p>Hollow Cylinder</p>	
<b>Description:</b>	<b>Equations:</b>
<b>Inertia (known weight)</b>	$J = (W \times (r_o^2 + r_i^2)) \div (2g)$
<b>Inertia (known density)</b>	$J = (\pi \times L \times \rho \times (r_o^4 - r_i^4)) \div (2g)$
<b>Volume</b>	$\text{volume} = \pi/4 \times (D_o^2 - D_i^2) \times L$
<i>(table continued next page)</i>	

Table 1 – Calculate the Torque for “Acceleration” and “Running” (continued from previous page)	
<b>Inertia of Solid Cylinder Equations</b>	
Solid Cylinder 	
<b>Description:</b>	<b>Equations:</b>
<b>Inertia (known weight)</b>	$J = (W \times r^2) \div (2g)$
<b>Inertia (known density)</b>	$J = (\pi \times L \times \rho \times r^4) \div (2g)$
<b>Volume</b>	volume = $\pi \times r^2 \times L$
<b>Inertia of Rectangular Block Equations</b>	
Rectangular Block 	
<b>Description:</b>	<b>Equations:</b>
<b>Inertia (known weight)</b>	$J = (W \div 12g) \times (h^2 + w^2)$
<b>Volume</b>	volume = $l \times h \times w$
<b>Symbol Definitions</b>	
<b>J = inertia, lb-in-s<sup>2</sup> (kg-m-s<sup>2</sup>)</b>	<b>ρ = density</b>
<b>L = Length, inches (m)</b>	<b>ρ = 0.098 lb/in<sup>3</sup> (aluminum)</b>
<b>h = height, inches (m)</b>	<b>ρ = 0.28 lb/in<sup>3</sup> (steel)</b>
<b>w = width, inches (m)</b>	<b>ρ = 0.04 lb/in<sup>3</sup> (plastic)</b>
<b>W = weight, lb (kg)</b>	<b>ρ = 0.31 lb/in<sup>3</sup> (brass)</b>
<b>D = diameter, inches (m)</b>	<b>ρ = 0.322 lb/in<sup>3</sup> (copper)</b>
<b>r = radius, inches (m)</b>	
<b>g = gravity = 386 in/sec<sup>2</sup> (9.8 m/s<sup>2</sup>)</b>	<b>π ≈ 3.14</b>

**LEADSCREW – EXAMPLE CALCULATIONS**

**STEP 1 – DEFINE THE ACTUATOR AND MOTION REQUIREMENTS**



- Weight of table and workpiece = 150 lb
- Angle of inclination = 0°
- Friction coefficient of sliding surfaces = 0.05
- External load force = 0
- Ball screw shaft diameter = 0.8 inch
- Ball screw length = 96 inch
- Ball screw material = steel
- Ball screw lead = 8.0 inch/rev ( $P \approx 0.125$  rev/in)
- Desired Resolution = 0.0005 inches/count
- Gear reducer = 2:1
- Stroke = 78 inches
- Move time = 12 seconds

Definitions
$d_{load}$ = lead or distance the load moves per revolution of the actuator's drive shaft ( $P = \text{pitch} = 1/d_{load}$ )
$D_{total}$ = total move distance
$\theta_{count}$ = servo resolution (counts/rev <sub>motor</sub> )
$i$ = gear reduction ratio (rev <sub>motor</sub> /rev <sub>gearshaft</sub> )
$T_{accel}$ = motor torque required to accelerate and decelerate the total system inertia (including motor inertia)
$T_{run}$ = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.
$t_{total}$ = move time

**STEP 2 – DETERMINE THE POSITIONING RESOLUTION OF THE LOAD**

The resolution of the load can be determined using **Equation 2**. If the servo motor is connected directly to the ballscrew, then the best resolution possible would be:

$$\begin{aligned}
 L_{\theta} &= (d_{load} \div i) \div \theta_{count} \\
 &= (8 \div 1) \div 10,000 \\
 &= 0.0008
 \end{aligned}$$

This does not meet the system requirements; however, if we add a 2:1 transmission with no lost motion (backlash, etc.) to the output of the motor, the resolution gets better by a factor of 2, so the minimum requirements would be met.

$$\begin{aligned}
 L_{\theta} &= (8 \div 2) \div 10,000 \\
 &= 0.0004
 \end{aligned}$$



**STEP 3 – DETERMINE THE MOTION PROFILE**

From **Equation 1**, the total pulses to make the required move is:

$$P_{total} = (D_{total} \div (d_{load} \div i)) \times \theta_{count}$$

$$= (78 \div (8 \div 2)) \times 10,000 = 195,000 \text{ pulses}$$

From **Equation 4**, the indexing frequency for a trapezoidal move is:

$$f_{TRAP} = (P_{total} - (f_{start} \times t_{ramp})) \div (t_{total} - t_{ramp})$$

$$= (195,000 - (100 \times 0.6)) \div (12 - 0.6) \approx 17.1 \text{ kHz}$$

where accel time is 5% of total move time and starting speed is 100Hz.

$$= 17.1 \text{ kHz} \times (60 \text{ sec}/1 \text{ min}) \div 10,000 \text{ counts/rev}$$

$$\approx 103 \text{ rpm}$$

**STEP 4 – DETERMINE THE REQUIRED MOTOR TORQUE**

Using the equations in **Table 1**:

$$J_{total} = J_{motor} + J_{gear} + ((J_{coupling} + J_{screw} + JW) \div i^2)$$

For this example, let's assume the gearbox and coupling inertia are zero.

$$J_W = (W \div (g \times e)) \times (1 \div 2\pi P)^2$$

$$= (150 \div (386 \times 0.9)) \times (1 \div 2 \times 3.14 \times 0.125)^2$$

$$\approx 0.700 \text{ lb}\cdot\text{in}\cdot\text{sec}^2$$

$$J_{screw} \approx (\pi \times L \times \rho \times r^4) \div (2g)$$

$$\approx (3.14 \times 96 \times 0.28 \times 0.0256) \div (2 \times 386)$$

$$\approx 0.0028 \text{ lb}\cdot\text{in}\cdot\text{sec}^2$$

The inertia of the load and screw reflected to the motor is:

$$J_{(screw + load) \text{ to motor}} = ((J_{screw} + J_W) \div i^2)$$

$$\approx ((0.0028 + 0.700) \div 2^2) = 0.176 \text{ lb}\cdot\text{in}\cdot\text{sec}^2$$

The torque required to accelerate the inertia is:

$$T_{accel} \approx J_{total} \times (\Delta_{speed} \div \Delta_{time}) \times 0.1$$

$$= 0.176 \times (103 \div 0.6) \times 0.1 \approx 1.08 \text{ lb}\cdot\text{in}$$

Next, we need to determine running torque. If the machine already exists then it is sometimes possible to actually measure running torque by turning the actuator driveshaft with a torque wrench.

$$T_{run} = ((F_{total} \div (2 \pi P)) + T_{preload}) \div i$$

$$F_{total} = F_{ext} + F_{friction} + F_{gravity}$$

$$= 0 + \mu W \cos\theta + 0 = 0.05 \times 150 = 7.5 \text{ lb}$$

$$T_{run} = (7.5 \div (2 \times 3.14 \times 0.125)) \div 2$$

$$\approx 4.77 \text{ lb}\cdot\text{in}$$

where we have assumed preload torque to be zero.

From **Equation 5**, the required motor torque is:

$$T_{motor} = T_{accel} + T_{run} = 1.08 + 4.77 \approx 5.85 \text{ lb}\cdot\text{in} \approx 0.66 \text{ N}\cdot\text{m}$$

However, this is the required motor torque before we have picked a motor and included the motor inertia.

**STEP 5 – SELECT AND CONFIRM THE SERVO MOTOR AND DRIVER SYSTEM**

It looks like a reasonable choice for a motor would be the SVL-207. This motor has an inertia of:

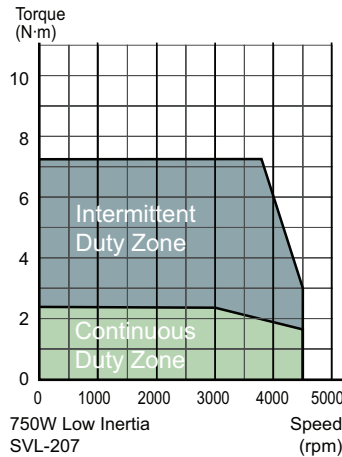
- $J_{motor} = 0.00096 \text{ lb}\cdot\text{in}\cdot\text{sec}^2$

The actual motor torque would be modified:

- $T_{accel} = J_{total} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$   
 $= (0.176 + 0.00096) \times (103 \div 0.6) \times 0.1$   
 $\approx 1.09 \text{ lb}\cdot\text{in}$

So that:

- $T_{motor} = T_{accel} + T_{run}$   
 $= 1.09 + 4.77 \approx 5.86 \text{ lb}\cdot\text{in} \approx 0.66 \text{ N}\cdot\text{m}$



It looks like the 750W system will work. However, we still need to check the load to motor inertia ratio:

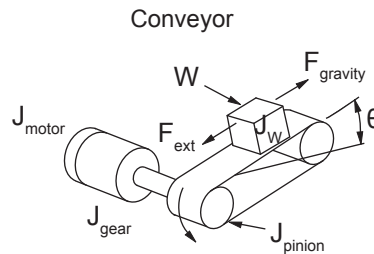
- $Ratio = J_{(screw + load) \text{ to motor}} \div J_{motor}$   
 $= 0.176 \div 0.00096 = 183.3$

It is best to keep the load to motor inertia ratio below 10, so 183 is well outside this guideline. Although the servo has enough power to control the system, the large mismatch ratio may prevent proper tuning and faster acceleration settings in the future. Since the motor speed required to move the system is well within the motor specs, we can change the gear ratio to use a 750W motor or select a much larger motor such as the SVM-220. Because the reflected inertia is decreased by the square of the ratio, we will change the gear ratio to 10:1. By doing this, the mismatch ratio is now 7.3 (before we consider any added inertia due to the reducer).

- $Reflected J = (J_{screw} + J_{load}) \div 2^2 = 0.176$ , so:
- $New Reflected J = (J_{screw} + J_{load}) \div 10^2 = 0.00704$
- $New J Ratios = 0.00704 \div 0.00096 = 7.33$

## BELT DRIVE – EXAMPLE CALCULATIONS

### STEP 1 – DEFINE THE ACTUATOR AND MOTION REQUIREMENTS



- Weight of table and workpiece = 90 lb
- External force = 0 lb
- Friction coefficient of sliding surfaces = 0.05
- Angle of table = 0°
- Belt and pulley efficiency = 0.8
- Pulley diameter = 2.0 inch
- Pulley thickness = 0.75 inch
- Pulley material = aluminum
- Desired Resolution = 0.0005 inch/step
- Gear Reducer = 10:1
- Stroke = 50 inch
- Move time = 4.0 seconds
- Accel and decel time = 1.0 seconds

Definitions
$d_{load}$ = lead or distance the load moves per revolution of the actuator's drive shaft ( $P = \text{pitch} = 1/d_{load}$ )
$D_{total}$ = total move distance
$\theta_{count}$ = servo resolution (counts/rev <sub>motor</sub> )
$i$ = gear reduction ratio (rev <sub>motor</sub> /rev <sub>gearshaft</sub> )
$T_{accel}$ = motor torque required to accelerate and decelerate the total system inertia (including motor inertia)
$T_{run}$ = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.
$t_{total}$ = move time

### STEP 2 – DETERMINE THE POSITIONING RESOLUTION OF THE LOAD

The resolution of the load can be determined using **Equation 2**. If the servo motor is connected directly to the pulley, then the best resolution possible would be:

$$\begin{aligned}
 L_{\theta} &= (d_{load} \div i) \div \theta_{count} \\
 &= ((\pi \times 2.0) \div 1) \div 10,000 \\
 &= 0.00063
 \end{aligned}$$

where  $d_{load} = \pi \times \text{Pulley Diameter}$ .

This does not meet the system requirements. However, if we add a 10:1 transmission to the output of the motor, the resolution improves by a factor of 10, meeting the minimum system requirements.

$$\begin{aligned}
 L_{\theta} &= ((\pi \times 2.0) \div 10) \div 10,000 \\
 &= 0.000063
 \end{aligned}$$

**STEP 3 – DETERMINE THE MOTION PROFILE**

From **Equation 1**, the total pulses to make the required move is:

$$\begin{aligned} P_{total} &= (D_{total} \div (d_{load} \div i)) \times \theta_{count} \\ &= 50 \div ((3.14 \times 2.0) \div 10 \times 10,000) \\ &\approx 795,775 \text{ pulses} \end{aligned}$$

From **Equation 4**, the running frequency for a trapezoidal move is:

$$\begin{aligned} f_{TRAP} &= (P_{total} - (f_{start} \times t_{ramp})) \div (t_{total} - t_{ramp}) \\ &= 795,775 \div (4 - 1) \\ &\approx 265,258 \text{ Hz or } 265.3 \text{ kHz} \\ &\quad [\text{where accel time is 25\% of total move time and starting speed is zero}] \\ &= 265.3 \text{ kHz} \times (60 \text{ sec}/1 \text{ min}) \div 10,000 \text{ counts/rev} \\ &\approx 1,592 \text{ rpm motor speed} \end{aligned}$$

**STEP 4 – DETERMINE THE REQUIRED MOTOR TORQUE**

Using the equations in **Table 1**:

$$J_{total} = J_{motor} + J_{gear} + ((J_{pulleys} + J_W) \div i^2)$$

For this example, let's assume the gearbox inertia is zero.

$$\begin{aligned} J_W &= (W \div (g \times e)) \times r^2 \\ &= (90 \div (386 \times 0.8)) \times 1 \\ &\approx 0.291 \text{ lb}\cdot\text{in}\cdot\text{sec}^2 \end{aligned}$$

Pulley inertia (remember, there are two pulleys) can be calculated as:

$$\begin{aligned} J_{pulleys} &\approx ((\pi \times L \times \rho \times r^4) \div (2g)) \times 2 \\ &\approx ((3.14 \times 0.75 \times 0.098 \times 1) \div (2 \times 386)) \times 2 \\ &\approx 0.0006 \text{ lb}\cdot\text{in}\cdot\text{sec}^2 \end{aligned}$$

The inertia of the load and pulleys reflected to the motor is:

$$\begin{aligned} J_{(pulleys + load) \text{ to motor}} &= ((J_{pulleys} + J_W) \div i^2) \\ &\approx ((0.291 + 0.0006) \div 100) \approx 0.0029 \text{ lb}\cdot\text{in}\cdot\text{sec}^2 \end{aligned}$$

The torque required to accelerate the inertia is:

$$\begin{aligned} T_{acc} &\approx J_{total} \times (\Delta_{speed} \div \Delta_{time}) \times 0.1 \\ &\approx 0.0029 \times (1592 \div 1) \times 0.1 \\ &\approx 0.46 \text{ lb}\cdot\text{in} \\ T_{run} &= (F_{total} \times r) \div i \\ F_{total} &= F_{ext} + F_{friction} + F_{gravity} \\ &= 0 + \mu W \cos\theta + 0 = 0.05 \times 100 = 5.0 \text{ lb} \\ T_{run} &= (5.0 \times 1) \div 10 \\ &\approx 0.50 \text{ lb}\cdot\text{in} \end{aligned}$$

From **Equation 5**, the required motor torque is:

$$T_{motor} = T_{accel} + T_{run} = 0.46 + 0.50 \approx 0.96 \text{ lb}\cdot\text{in} \approx 0.11 \text{ N}\cdot\text{m}$$

However, this is the required motor torque before we have picked a motor and included the motor inertia.

**STEP 5 – SELECT AND CONFIRM THE SERVO MOTOR AND DRIVER SYSTEM**

It looks like a reasonable choice for a motor would be the SVL-2040. This motor has an inertia of:

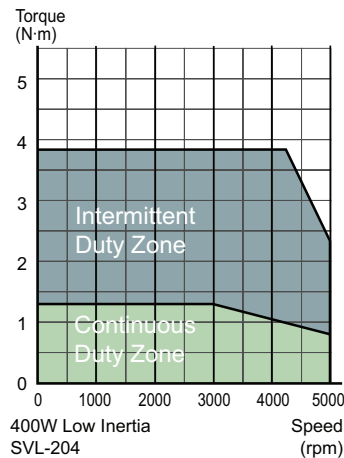
- $J_{motor} = 0.0003 \text{ lb}\cdot\text{in}\cdot\text{sec}^2$

The actual motor torque would be modified:

- $T_{accel} = J_{total} \times (\Delta_{speed} \div \Delta_{time}) \times 0.1$   
 $= (0.0029 + 0.0003) \times (1592 \div 1) \times 0.1 \approx 0.51 \text{ lb}\cdot\text{in}$

So that:

- $T_{motor} = T_{accel} + T_{run}$   
 $= 0.51 + 0.5 \approx 1.01 \text{ lb}\cdot\text{in} \approx 0.12 \text{ N}\cdot\text{m}$



It looks like the 400W system will work. However, we still need to check the load to motor inertia ratio:

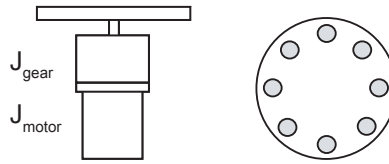
- $Ratio = J_{(pulleys + load) \text{ to motor}} \div J_{motor}$   
 $= 0.0029 \div 0.0003 = 9.6$

It is best to keep the load to motor inertia ratio at or below 10, so 9.6 is within an acceptable range.

**INDEX TABLE – EXAMPLE CALCULATIONS**

**STEP 1 – DEFINE THE ACTUATOR AND MOTION REQUIREMENTS**

Index Table



- Diameter of index table = 12 inch
- Thickness of index table = 3.25 inch
- Table material = steel
- Number of workpieces = 8
- Desired Resolution = 0.006°
- Gear Reducer = 6:1
- Index angle = 45°
- Index time = 0.5 seconds

Definitions
$d_{load}$ = lead or distance the load moves per revolution of the actuator's drive shaft ( $P = \text{pitch} = 1/d_{load}$ )
$D_{total}$ = total move distance
$\theta_{count}$ = servo resolution (counts/rev <sub>motor</sub> )
$i$ = gear reduction ratio (rev <sub>motor</sub> /rev <sub>gearshaft</sub> )
$T_{accel}$ = motor torque required to accelerate and decelerate the total system inertia (including motor inertia)
$T_{run}$ = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.
$t_{total}$ = move time

**STEP 2 – DETERMINE THE POSITIONING RESOLUTION OF THE LOAD**

The resolution of the load can be determined using **Equation 4**. If the servo motor is connected directly to the table, then the best resolution possible would be:

$$\begin{aligned}
 L_{\theta} &= (d_{load} \div i) \div \theta_{count} \\
 &= (360^{\circ} \div 1) \div 10,000 \\
 &= 0.036^{\circ}
 \end{aligned}$$

This does not meet the system requirements. However, if we add a 6:1 transmission to the output of the motor, the resolution gets better by a factor of 6, meeting the minimum system requirements.

$$\begin{aligned}
 L_{\theta} &= (360^{\circ} \div 6) \div 10,000 \\
 &= 0.006^{\circ}
 \end{aligned}$$

**STEP 3 – DETERMINE THE MOTION PROFILE**

From **Equation 1**, the total pulses to make the required move is:

$$\begin{aligned}
 P_{total} &= (D_{total} \div (d_{load} \div i)) \times \theta_{count} \\
 &= (45^{\circ} \div (360^{\circ} \div 6)) \times 10,000 \\
 &= 7,500 \text{ pulses}
 \end{aligned}$$

From **Equation 4**, the running frequency for a trapezoidal move is:

$$\begin{aligned}
 f_{TRAP} &= (P_{total} - (f_{start} \times t_{ramp})) \div (t_{total} - t_{ramp}) \\
 &= 7,500 \div (0.5 - 0.13) \approx 20.27 \text{ kHz} \\
 &\quad [\text{where accel time is 25\% of total move time and starting speed is zero}] \\
 &= 20.27 \text{ kHz} \times (60 \text{ sec}/1 \text{ min}) \div 10,000 \text{ counts/rev} \\
 &\approx 121 \text{ rpm}
 \end{aligned}$$

**STEP 4 – DETERMINE THE REQUIRED MOTOR TORQUE**

Using the equations in **Table 1**:

- $J_{total} = J_{motor} + J_{gear} + (J_{table} \div i^2)$

For this example, let's assume the gearbox inertia is zero.

- $J_{table} \approx (\pi \times L \times \rho \times r^4) \div (2g)$   
 $\approx (3.14 \times 3.25 \times 0.28 \times 1296) \div (2 \times 386)$   
 $\approx 4.80 \text{ lb}\cdot\text{in}\cdot\text{sec}^2$

The inertia of the indexing table reflected to the motor is:

- $J_{table \text{ to motor}} = J_{table} \div i^2$   
 $\approx 0.133 \text{ lb}\cdot\text{in}\cdot\text{sec}^2$

The torque required to accelerate the inertia is:

- $T_{accel} \approx J_{total} \times (\Delta_{speed} \div \Delta_{time}) \times 0.1$   
 $= 0.133 \times (121 \div 0.13) \times 0.1$   
 $\approx 12.38 \text{ lb}\cdot\text{in}$

From **Equation 5**, the required motor torque is:

- $T_{motor} = T_{accel} + T_{run}$   
 $= 12.38 + 0 = 12.38 \text{ lb}\cdot\text{in} \approx 1.40 \text{ N}\cdot\text{m}$

However, this is the required motor torque before we have picked a motor and included the motor inertia.

**STEP 5 – SELECT AND CONFIRM THE SERVO MOTOR AND DRIVER SYSTEM**

It looks like a reasonable choice for a motor would be the SVM-220. This motor has an inertia of:

- $J_{motor} = 0.014 \text{ lb}\cdot\text{in}\cdot\text{sec}^2$

The actual motor torque would be modified:

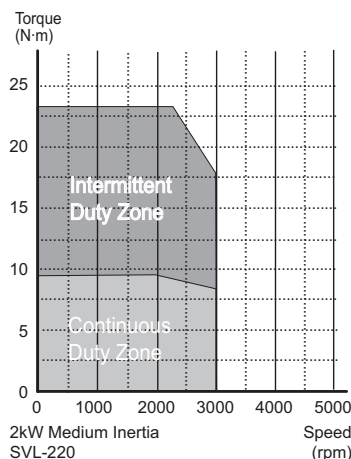
$$T_{accel} = J_{total} \times (\Delta_{speed} \div \Delta_{time}) \times 0.1$$

$$= (0.133 + 0.014) \times (121 \div 0.13) \times 0.1$$

$$\approx 13.68 \text{ lb}\cdot\text{in}$$

So that:

- $T_{motor} = T_{accel} + T_{run}$   
 $= 13.68 + 0$   
 $= 13.68 \text{ lb}\cdot\text{in} \approx 1.55 \text{ N}\cdot\text{m}$



It looks like the 2 kW medium inertia system will work. However, we still need to check the load to motor inertia ratio:

- $Ratio = J_{table \text{ to motor}} \div J_{motor}$   
 $= 0.133 \div 0.014 = 9.5$

It is best to keep the load to motor inertia ratio at or below 10, so 9.5 is within an acceptable range.

## ENGINEERING UNIT CONVERSION TABLES, FORMULAS, &amp; DEFINITIONS

Conversion of Length							
*	B						
	$\mu\text{m}$	mm	m	mil	in	ft	
A	$\mu\text{m}$	1	1.000E-03	1.000E-06	3.937E-02	3.937E-05	3.281E-06
	mm	1.000E+03	1	1.000E-03	3.937E+01	3.937E-02	3.281E-03
	m	1.000E+06	1.000E+03	1	3.937E+04	3.937E+01	3.281E+00
	mil	2.540E+01	2.540E-02	2.540E-05	1	1.000E-03	8.330E-05
	in	2.540E+04	2.540E+01	2.540E-02	1.000E+03	1	8.330E-02
	ft	3.048E+05	3.048E+02	3.048E-01	1.200E+04	1.200E+01	1

\* To convert A to B, multiply A by the entry in the table.

Conversion of Torque							
*	B						
	N-m	kpm(kg-m)	kg-cm	oz-in	lb-in	lb-ft	
A	N-m	1	1.020E-01	1.020E+01	1.416E+02	8.850E+00	7.380E-01
	kg-m (kpm)	9.810E+00	1	1.000E+02	1.390E+03	8.680E+01	7.230E+00
	kg-cm	9.810E-02	1.000E-02	1	1.390E+01	8.680E-01	7.230E-02
	oz-in	7.060E-03	7.200E-04	7.200E-02	1	6.250E-02	5.200E-03
	lb-in	1.130E-01	1.150E-02	1.150E+00	1.600E+01	1	8.330E-02
	lb-ft	1.356E+00	1.380E-01	1.383E+01	1.920E+02	1.200E+01	1

\* To convert A to B, multiply A by the entry in the table.

Conversion of Moment of Inertia								
*	B							
	kg-m <sup>2</sup>	kg-cm-s <sup>2</sup>	oz-in-s <sup>2</sup>	lb-in-s <sup>2</sup>	oz-in <sup>2</sup>	lb-in <sup>2</sup>	lb-ft <sup>2</sup>	
A	kg-m <sup>2</sup>	1	1.020E+01	1.416E+02	8.850E+00	5.470E+04	3.420E+03	2.373E+01
	kg-cm-s <sup>2</sup>	9.800E-02	1	1.388E+01	8.680E-01	5.360E+03	3.350E+02	2.320E+00
	oz-in-s <sup>2</sup>	7.060E-03	7.190E-02	1	6.250E-02	3.861E+02	2.413E+01	1.676E-01
	lb-in-s <sup>2</sup>	1.130E-01	1.152E+00	1.600E+01	1	6.180E+03	3.861E+02	2.681E+00
	oz-in <sup>2</sup>	1.830E-05	1.870E-04	2.590E-03	1.620E-04	1	6.250E-02	4.340E-04
	lb-in <sup>2</sup>	2.930E-04	2.985E-03	4.140E-02	2.590E-03	1.600E+01	1	6.940E-03
	lb-ft <sup>2</sup>	4.210E-02	4.290E-01	5.968E+00	3.730E-01	2.304E+03	1.440E+02	1

\* To convert A to B, multiply A by the entry in the table.

General Formulae & Definitions	
Description:	Equations:
Gravity	gravity = 9.8 m/s <sup>2</sup> = 386 in/s <sup>2</sup>
Torque	T = J $\alpha$ , $\alpha$ = rad/s <sup>2</sup>
Power (Watts)	P(W) = T(N-m) · $\omega$ (rad/s)
Power (Horsepower)	P(hp) = T(lb-in) · v(rpm) / 63,024
Horsepower	1 hp = 746 W
Revolutions	1 rev = 1,296,000 arc-sec = 21,600 arc-min = 360 degrees

Equations for Straight-Line Velocity & Constant Acceleration	
Description:	Equations:
Final velocity	$v_f = v_i + at$ final velocity = initial velocity + (acceleration · time)
Final position	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$ final position = initial position + [ $\frac{1}{2}$ · (initial velocity + final velocity) · time]
Final position	$x_f = x_i + v_i t + \frac{1}{2}at^2$ final position = initial position + (initial velocity · time) + ( $\frac{1}{2}$ · acceleration · time squared)
Final velocity squared	$v_f^2 = v_i^2 + 2a(x_f - x_i)$ final velocity squared = initial velocity squared + [2·acceleration·(final position - initial position)]